

Isospin Effects of the Critical Behavior in the Lattice Gas Model *

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Isospin effects of the critical phenomena were studied via Xe isotopes in the frame of lattice gas model. All the critical temperatures for four Xe isotopes are close to 5.5 MeV at the same freeze-out density of about $0.39 \rho_0$. The critical values of power law parameter of mass distribution, mean multiplicity of intermediate mass fragments (IMF), information entropy and Campi's second moment show minor dependence on the isospin at the critical point.

With the development of the accelerator and radioactive beam technique, a lot of new experiments using the radioactive beams with large neutron or proton excess become possible. The degree of freedom of isospin of nuclear matter is becoming important for research. It offers the possibility to study the properties of nuclear matter in the range from symmetrical nuclear matter to pure neutron matter. Some theoretical investigations to the equation of state, chemical and mechanical instabilities as well as liquid-gas phase transition for isospin asymmetrical nuclear matter were performed already. In addition, the isospin dependent nucleon - nucleon cross section is also an important subject due to its significant effects on the dynamical process of heavy ion reactions induced by radioactive beams. Some new phenomena stemmed from the isospin have been revealed. For examples, the isospin dependences of the preequilibrium nucleon emission, the nuclear stopping, the nuclear collective flow, total reaction cross section, radii of neutron-rich nuclei and subthreshold pion production have been studied by several groups [1-3]. However, more experimental and theoretical studies are still needed for understanding the isospin physics. As a trial, the isospin effects were investigated with the lattice gas model in this letter.

The lattice gas model of Lee and Yang [4], in which the grandcanonical partition function of a gas with one type of atoms is mapped into the canonical ensemble of an Ising model for spin 1/2 particles, has successfully described the liquid-gas phase transition for atomic system. The same model has already been applied to nuclear physics for isospin symmetrical systems in the grandcanonical ensemble [5] with an approximate sampling [6] of the canonical ensemble [7-10], and also for isospin asymmetrical nuclear matter in the mean field approximation [11]. In this model, A nucleons with an occupation number s which is defined as $s = 1$ (-1) for a proton (neutron) or $s = 0$ for a vacancy, are placed in the L sites of lattice. Nucleons in the nearest neighbouring sites have interaction with an energy $\epsilon_{s_i s_j}$. The hamiltonian is written by

$$E = \sum_{i=1}^A \frac{p_i^2}{2m} - \sum_{i<j} \epsilon_{s_i s_j} s_i s_j \quad (1)$$

The interaction constant $\epsilon_{s_i s_j}$ is fixed to reproduce the binding energy of the nuclei, $\epsilon_{nn,pp} = \epsilon_{-1-1,11} = 0$. MeV, $\epsilon_{pn,np} = \epsilon_{1-1,-11} = -5.33$ MeV. We use a three-dimension cubic lattice L with a size l , a number of nucleons $A = N + Z$ and a temperature T . The freeze-out density of disassembling system is $\rho_f = \frac{A}{L} \rho_0$ where ρ_0 is the normal nucleon density. The disassembly of the system is to be calculated at ρ_f , beyond which nucleons are too far apart to interact. A nucleons are put in L cubes by Monte Carlo sampling using the Metropolis algorithm [12]. Once the nucleons have been placed, their momentum is generated by a Monte Carlo sampling of Maxwell Boltzmann distribution. Various observables can be calculated in a straightforward fashion.

One of the basic measurable quantities is the distribution of fragment mass. In this lattice gas model, two neighboring nucleons are viewed to be in the same fragment if their relative kinetic energy is insufficient to overcome the

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attractive bond: $P_r^2/2\mu + \epsilon_{s_i s_j} < 0$. This method is similar to the so-called Coniglio-Klein's prescription [13]. In this letter, we use the above condition to construct the fragments and their distributions.

We chose several isotopes of Xe as examples of the study of isospin effects in the lattice gas model. Their isospin parameter ($\frac{N-Z}{A}$) is 0.11, 0.16, 0.21 and 0.26 for ^{122}Xe , ^{129}Xe , ^{137}Xe and ^{146}Xe , respectively. The freeze-out density ρ_f has been chosen to be close to $0.39 \rho_0$, extracted from the analysis of Ar + Sc [7] and $^{35}\text{Cl} + \text{Au}$ and $^{70}\text{Ge} + \text{Ti}$ [14] with the same model. There is also good support from experiment that the value of ρ_f is significantly below $0.5\rho_0$ [15]. We use the 343 cubic lattice with size of 7 which results that the freeze-out density ρ_f/ρ_0 of $^{122,129,137,146}\text{Xe}$ is 0.36, 0.38, 0.40, and 0.43, respectively. The other input parameter is the temperature, we perform the calculation from 4 to 7 MeV. For each isotope 1000 events are accumulated at each temperature.

Fig.1 shows the mass distribution of fragments at $T = 4, 5, 6$ and 7 MeV for ^{129}Xe . Clearly the disassembling mechanism evolves with the nuclear temperature. A few light particles and fragments are emitted and the big residue reserves at $T = 4$ MeV which indicates typical evaporation mechanism. With the increasing temperature, the shoulder of mass distribution occurs due to the competition between the fragmentation and the evaporation. This shoulder disappears and the mass distribution becomes power law shape at $T = 6$ MeV, corresponding to the multifragmentation region. When the temperature becomes much higher, the mass distribution becomes steeper indicating that the disassembling process becomes more violent. The power law fit, $Y(A) \propto A^{-\tau}$, for these mass distribution can be introduced here. It has already been observed that a minimum of power law parameter τ_{min} exists for most systems if the critical behavior takes place. The lines in Fig.1 represent the power law fit. Fig.2 displays the several physical quantities as a function of temperature for Xe nuclei with the different isospin. The minimums of τ parameters in Fig.2a locate closely at 5.5 MeV for all the systems, which illustrates its minor dependence on the isospin. In other words, there is a universal mass distribution regardless of the size of disassembling source when the critical phenomenon takes place. However, the τ parameters show different values outside the critical region for nuclei with different isospin, eg., τ decreases with isospin when $T > 5.5$ MeV (multifragmentation region). Similarly, the mean multiplicity of intermediate mass fragment N_{IMF} , defined as the number of fragments with $3 \leq Z \leq 16$ here, has analogous characters in Fig.2b [17,18]. There are the maximums for Xe systems near to 5.5 MeV. When the temperature becomes higher, the larger the source, the higher the N_{IMF} .

Fig.2c plots the information entropy H as a function of temperature for Xe isotopes. The information entropy was introduced by Shannon in information theory first [19]. It is defined as

$$H = - \sum_i p_i \ln(p_i), \quad (2)$$

where p_i is the probability having "i" produced particles in each event, the sum is taken over all multiplicities of products from the disassembling system. H reflects the capacity of the information or the extent of disorder. We introduce this entropy into the nuclear disassembly here. As expected, the entropy H reveals the peak close to 5.5 MeV for all isotopes. These peaks indicate that the opening of the phase space and the number of the states at the critical point is the largest. In the other words, the systems at the critical point have the largest fluctuation which leads to the largest disorder. After the critical point, the entropy H increases with the isospin and/or the source size.

In Fig.2d we give the temperature dependences of Campi's second moment of the mass distribution [16], which is defined as

$$S_2 = \frac{\sum_{i \neq A_{max}} A_i^2 \times n_i(A_i)}{A}, \quad (3)$$

where $n_i(A_i)$ is the number of clusters with A_i nucleons and the sum excludes the largest cluster A_{max} . A is the mass of the system. At the percolation point S_2 diverges in an infinite system and is at maximum in a finite system. Fig.2d gives the maximums of S_2 around 5.5 MeV for different isotopes, respectively. Again, the critical behavior occurs in the same temperature as other observables.

In conclusion, the critical behaviors are explored for Xe isotopes in the lattice gas model, namely the minimum of power-law parameter τ of mass distribution, the rise and fall of mean multiplicity of IMF, information entropy and Campi's second moment. In a narrow region of critical point, the features of the above quantities show no dependence on the isospin of the disassembling system. It reflects that a universal law exists for the same element in the critical point. On the contrary, these quantities have isospin dependence at the same temperature outside the critical region. Noting that the information entropy is introduced into such an analysis for the first time, and it seems to be useful for the searching of critical phenomena in nuclear physics. It will be interesting and meaningful to have some experiments to compare our conclusion in the near future.

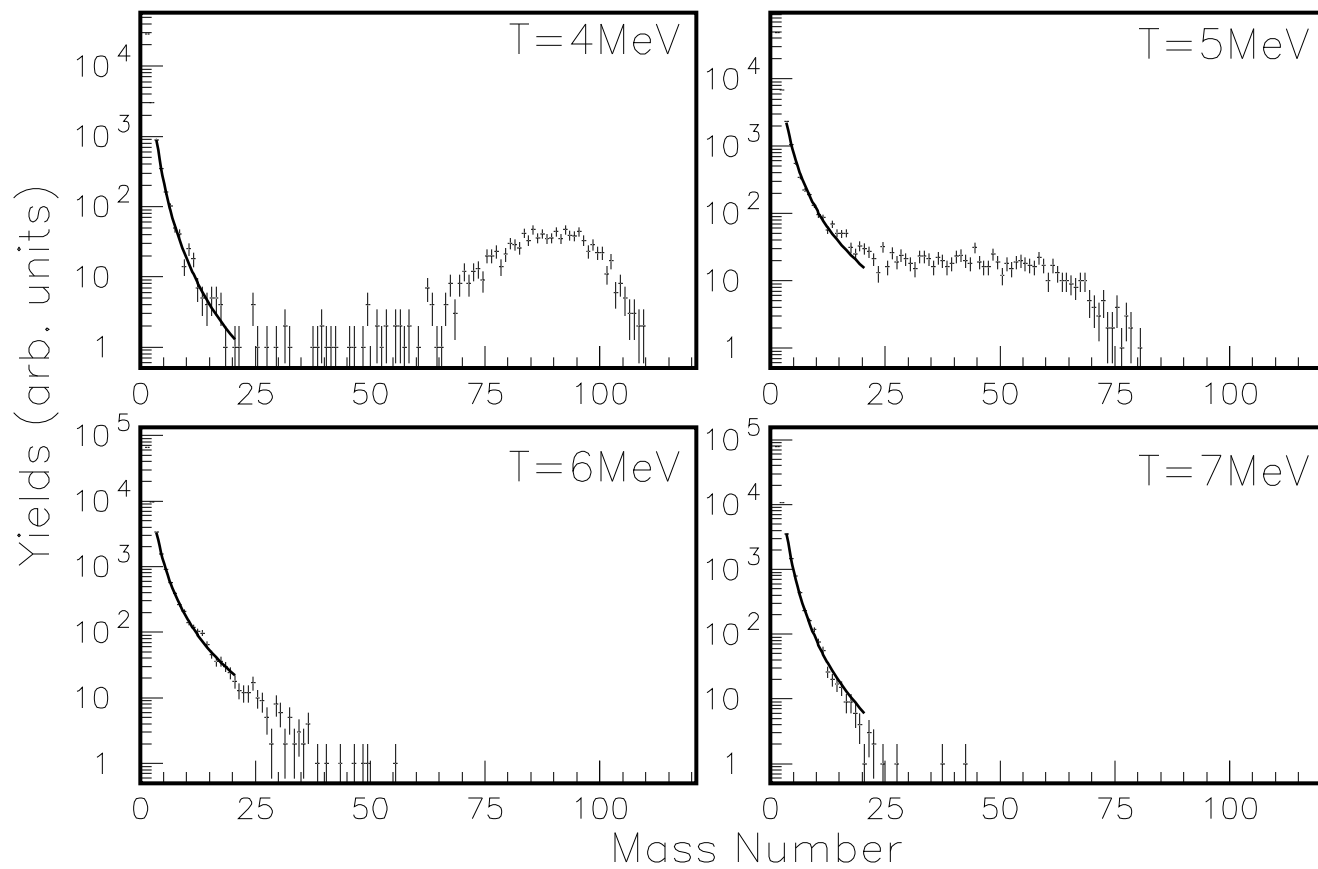
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Figure Captions

Fig.1: Mass distribution of ^{129}Xe at $T=4, 5, 6$ and 7 MeV. The lines are the power-law fit.

Fig.2: Critical observables: the τ parameter from the power law fit to mass distribution (a), the average multiplicity of intermediate mass fragments (b), the information entropy (c) and the Campi's second moment (d) as functions of temperature and isospin.



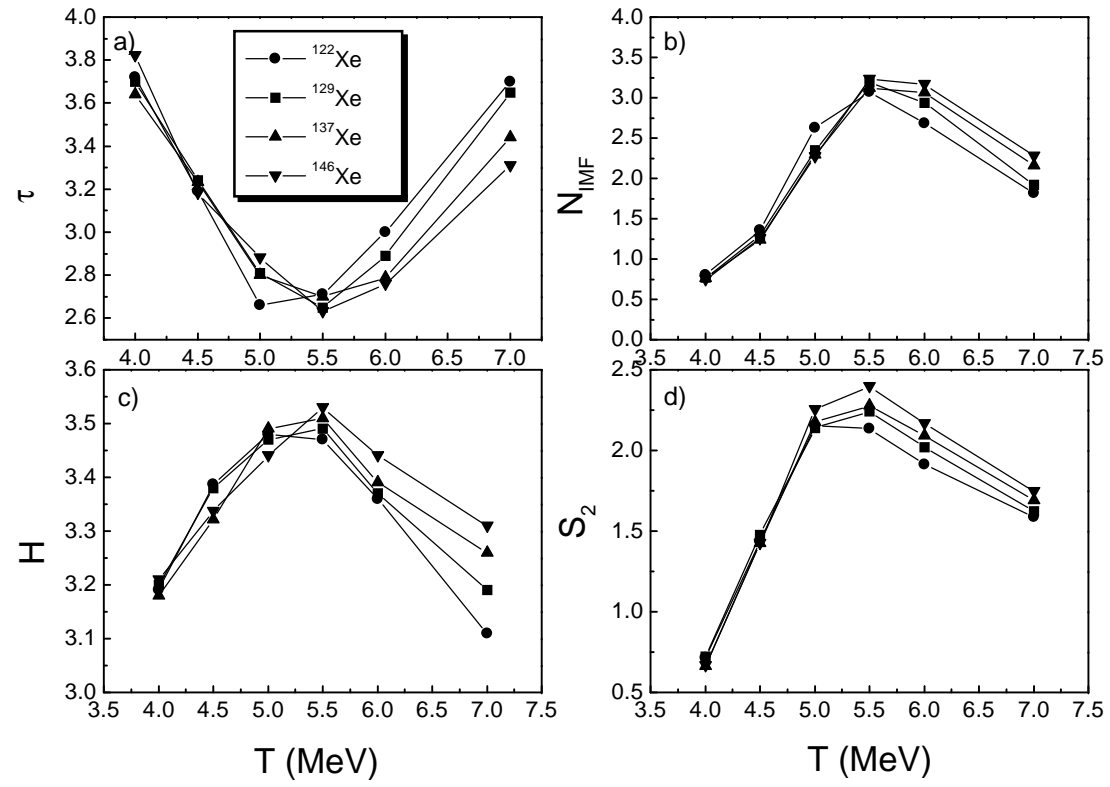


Fig.2